Midterm Review

Topics on the Midterm

- Data Structures & Object-Oriented Design
- Run-Time Analysis
- Linear Data Structures
- The Java Collections Framework
- Recursion
- Trees
- Priority Queues & Heaps

Data Structures So Far

- Array List
  - (Extendable) Array
- Node List
  - Singly or Doubly Linked List
- Stack
  - Array
  - Singly Linked List
- Queue
  - Array
  - Singly or Doubly Linked List
- Priority Queue
  - Unsorted doubly-linked list
  - Sorted doubly-linked list
  - Heap (array-based)
- Adaptable Priority Queue
  - Sorted doubly-linked list with location-aware entries
  - Heap with location-aware entries
- Tree
  - Linked Structure
- Binary Tree
  - Linked Structure
  - Array

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Data Structures & Object-Oriented Design

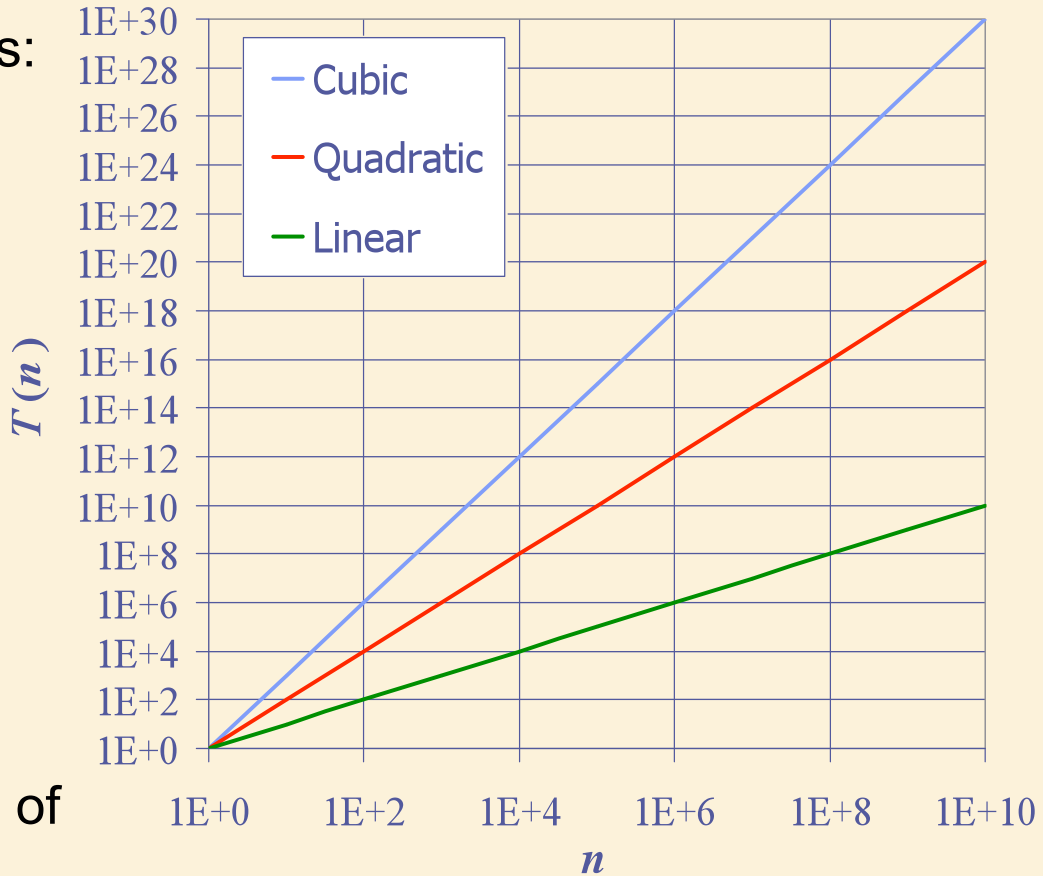
- Definitions
- Principles of Object-Oriented Design
- Hierarchical Design in Java
- Abstract Data Types & Interfaces
- Casting
- Generics
- Pseudo-Code

Software Engineering

- Software must be:
  - Readable and understandable
    - Allows correctness to be verified, and software to be easily updated.
  - Correct and complete
    - Works correctly for all expected inputs
  - Robust
    - Capable of handling unexpected inputs.
  - Adaptible
    - All programs evolve over time. Programs should be designed so that re-use, generalization and modification is easy.
  - Portable
    - Easily ported to new hardware or operating system platforms.
  - Efficient
    - Makes reasonable use of time and memory resources.

Seven Important Functions

- Seven functions that often appear in algorithm analysis:
  - Constant ≈ 1
  - Logarithmic ≈ $\log n$
  - Linear ≈ n
  - N-Log-N ≈ $n \log n$
  - Quadratic ≈ n^2
  - Cubic ≈ n^3
  - Exponential ≈ 2^n



- In a log-log chart, the slope of the line corresponds to the growth rate of the function.

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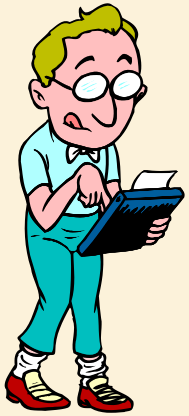
Some Math to Review



- Summations
- Logarithms and Exponents
- Existential and universal operators
- Proof techniques
- Basic probability

- **existential and universal operators**

$$\exists g \forall b \; Loves(b, g)$$

$$\forall g \exists b \; Loves(b, g)$$

- **properties of logarithms:**

$\log_b(xy) = \log_b x + \log_b y$

$\log_b (x/y) = \log_b x - \log_b y$

$\log_b x^a = a\log_b x$

$\log_b a = \log_x a / \log_x b$

- **properties of exponentials**:

$a^{(b+c)} = a^b a^c$

$a^{bc} = (a^b)^c$

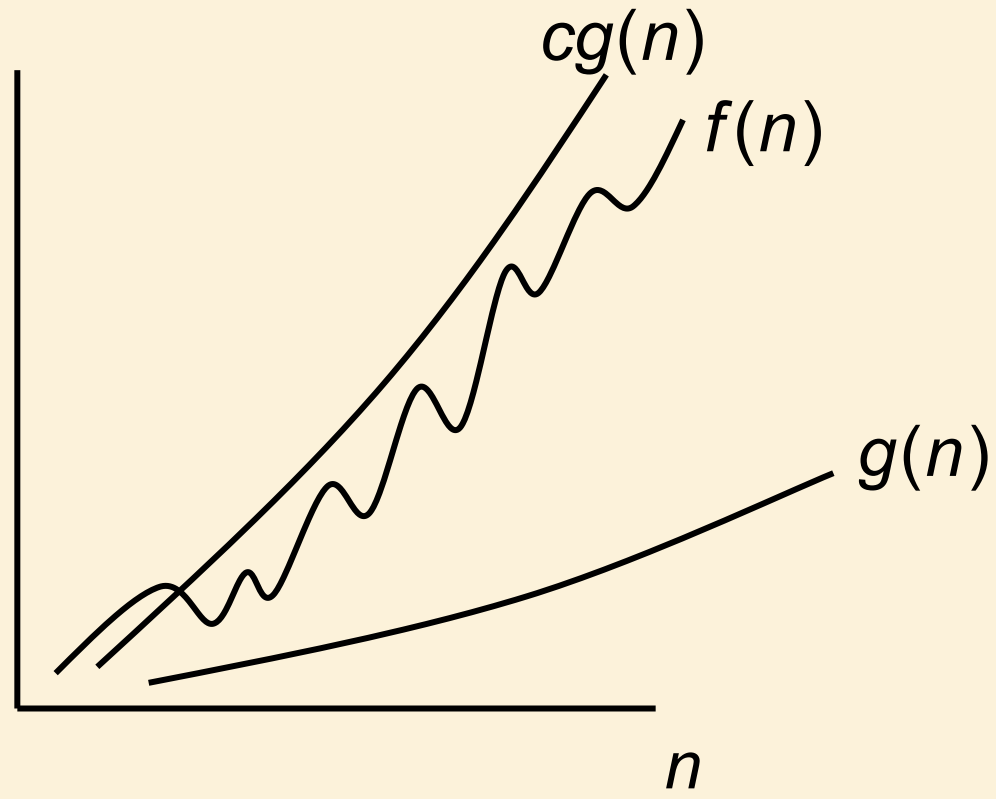
$a^b / a^c = a^{(b-c)}$

$b = a^{\log_a b}$

$b^c = a^{c^*\log_a b}$

Definition of "Big Oh"

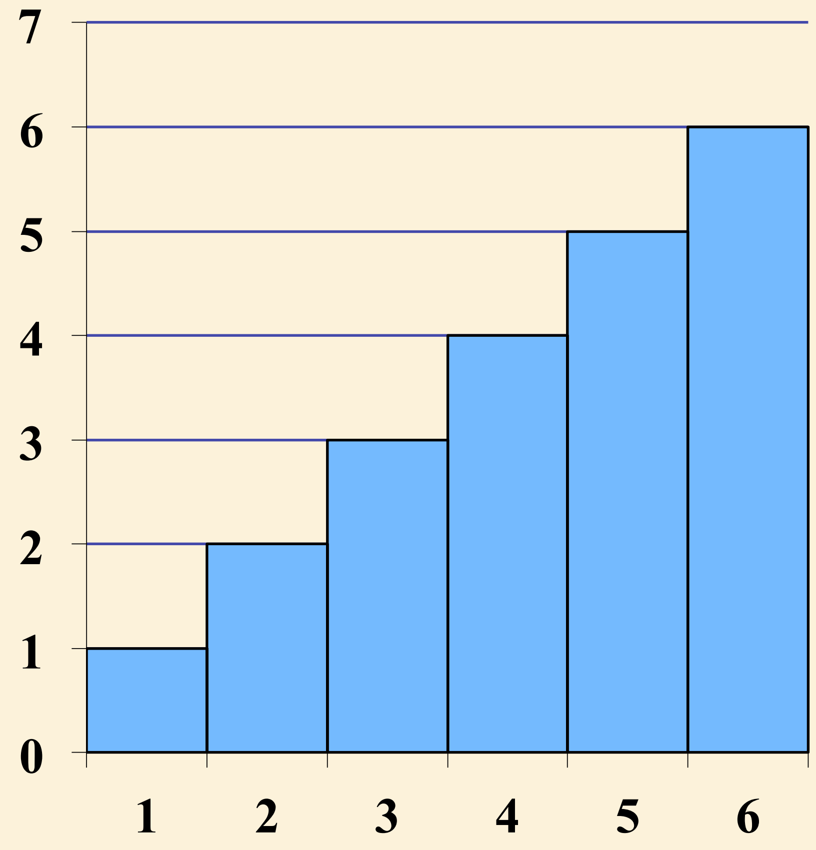
$$f(n) \in O(g(n))$$



$$\exists c, n_0 > 0 : \forall n \geq n_0, f(n) \leq cg(n)$$

Arithmetic Progression

- The running time of ***prefixAverages1*** is $O(1+2+\ldots+n)$
- The sum of the first n integers is $n(n+1)/2$
  - There is a simple visual proof of this fact
- Thus, algorithm ***prefixAverages1*** runs in $O(n^2)$ time



Relatives of Big-Oh

- **big-Omega**
  - f(n) is $\Omega(g(n))$ if there is a constant $c > 0$ and an integer constant $n_0 \geq 1$ such that $f(n) \geq c \cdot g(n)$ for $n \geq n_0$
- **big-Theta**
  - f(n) is $\Theta(g(n))$ if there are constants $c_1 > 0$ and $c_2 > 0$ and an integer constant $n_0 \geq 1$ such that $c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$ for $n \geq n_0$

Time Complexity of an Algorithm

The time complexity of an algorithm is the *largest* time required on *any* input of size n. (Worst case analysis.)

- $O(n^2)$: For any input size $n \geq n_0$, the algorithm takes no more than cn^2 time on every input.
- $\Omega(n^2)$: For any input size $n \geq n_0$, the algorithm takes at least cn^2 time on at least one input.
- $\theta(n^2)$: Do both.

Time Complexity of a Problem

The time complexity of a problem is the time complexity of the *fastest* algorithm that solves the problem.

- $O(n^2)$: Provide an algorithm that solves the problem in no more than this time.
  - Remember: for every input, i.e. worst case analysis!
- $\Omega(n^2)$: Prove that no algorithm can solve it faster.
  - Remember: only need one input that takes at least this long!
- $\theta(n^2)$: Do both.

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Arrays

Arrays

- Array: a sequence of indexed components with the following properties:
  - **array size is fixed** at the time of array's construction
    - **int**[] numbers = **new int** [10];
  - **array elements are placed contiguously** in memory
    - address of any element can be calculated directly as its offset from the beginning of the array
  - consequently, array components **can be efficiently inspected or updated** in O(1) time, using their indices
    - **randomNumber = numbers[5];**
    - **numbers[2] = 100;**

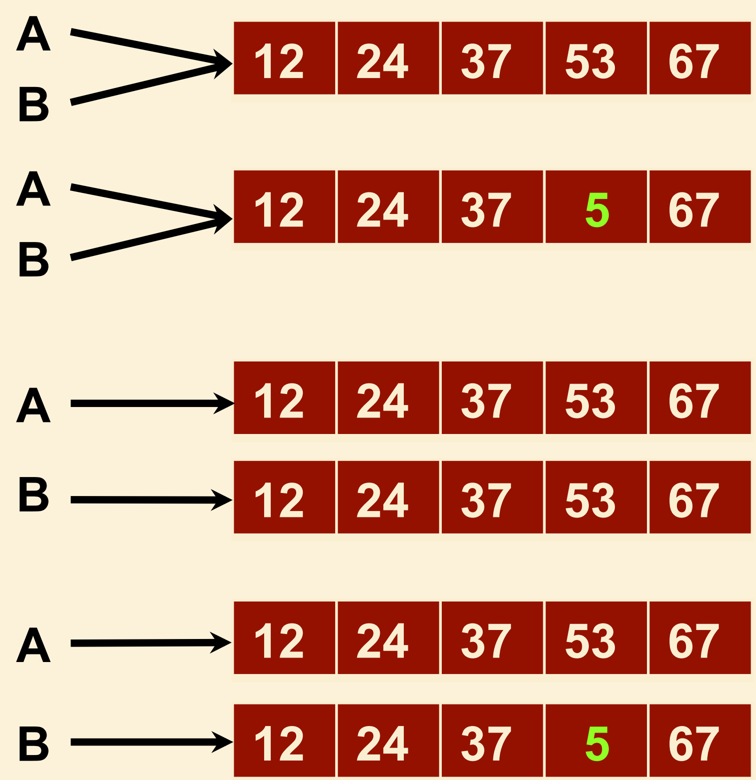
Arrays in Java

- Since an array is an object, the name of the array is actually a **reference** (pointer) to the place in memory where the array is stored.
  - reference to an object holds the **address** of the actual object
- Example [arrays as objects]

```
int[] A={12, 24, 37, 53, 67};
int[] B=A;
B[3]=5;
```

- Example [cloning an array]

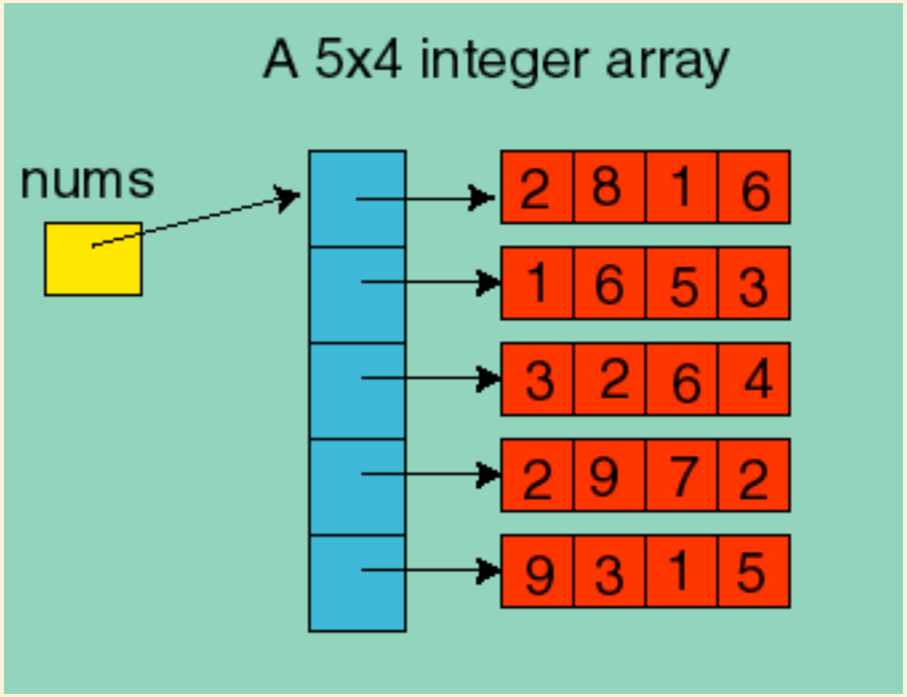
```
int[] A={12, 24, 37, 53, 67};
int[] B=A.clone();
B[3]=5;
```



Example

Example [2D array in Java = array of arrays]

```
int[][] nums = new int[5][4];

int[][] nums;

nums = new int[5][];

for (int i=0; i<5; i++) {

    nums[i] = new int[4];

}
```



Array Lists

The Array List ADT (§6.1)

- The Array List ADT extends the notion of array by storing a sequence of arbitrary objects
- An element can be accessed, inserted or removed by specifying its rank (number of elements preceding it)
- An exception is thrown if an incorrect rank is specified (e.g., a negative rank)

The Array List ADT

```
public interface IndexList<E> {
/** Returns the number of elements in this list */
public int size();
/** Returns whether the list is empty. */
public boolean isEmpty();
/** Inserts an element e to be at index I, shifting all elements after this. */
public void add(int I, E e) throws IndexOutOfBoundsException;
/** Returns the element at index I, without removing it. */
public E get(int i) throws IndexOutOfBoundsException;
/** Removes and returns the element at index I, shifting the elements after this. */
public E remove(int i) throws IndexOutOfBoundsException;
/** Replaces the element at index I with e, returning the previous element at i. */
public E set(int I, E e) throws IndexOutOfBoundsException;
}
```

Performance

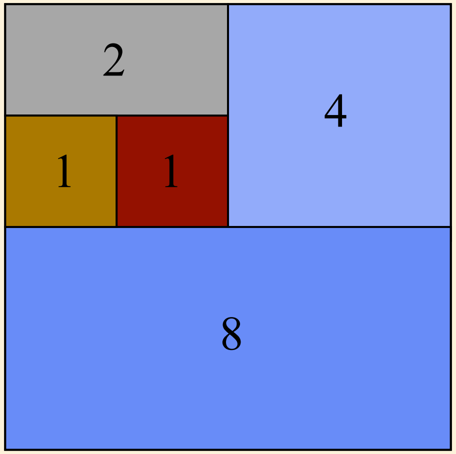
- In the array based implementation
  - The space used by the data structure is $O(n)$
  - ***size***, ***isEmpty***, ***get*** and ***set*** run in $O(1)$ time
  - ***add*** and ***remove*** run in $O(n)$ time
- In an ***add*** operation, when the array is full, instead of throwing an exception, we could replace the array with a larger one.
- In fact **java.util.ArrayList** implements this ADT using **extendable arrays** that do just this.

Doubling Strategy Analysis

- We replace the array $k = \log_2 n$ times
- The total time $T(n)$ of a series of n **add(o)** operations is proportional to

$$n + 1 + 2 + 4 + 8 + \ldots + 2^k = n + 2^{k+1} - 1 = 2n - 1$$ geometric series

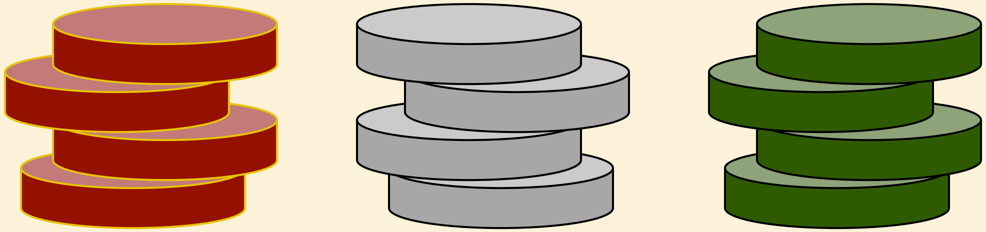
- Thus $T(n)$ is $O(n)$
- **The amortized time of an add operation is $O(1)$!**



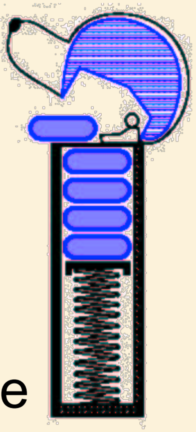
$$\left(\text{Recall: } \sum_{i=0}^{n} r^i = \frac{1 - r^{n+1}}{1 - r} \right)$$

Stacks

Chapter 5.1



The Stack ADT

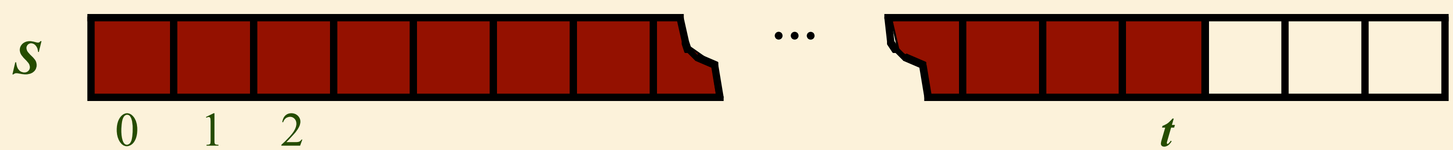


- The Stack ADT stores arbitrary objects
- Insertions and deletions follow the last-in first-out scheme
- Think of a spring-loaded plate dispenser
- Main stack operations:
  - push(object): inserts an element
  - object pop(): removes and returns the last inserted element
- Auxiliary stack operations:
  - object top(): returns the last inserted element without removing it
  - integer size(): returns the number of elements stored
  - boolean isEmpty(): indicates whether no elements are stored

Array-based Stack

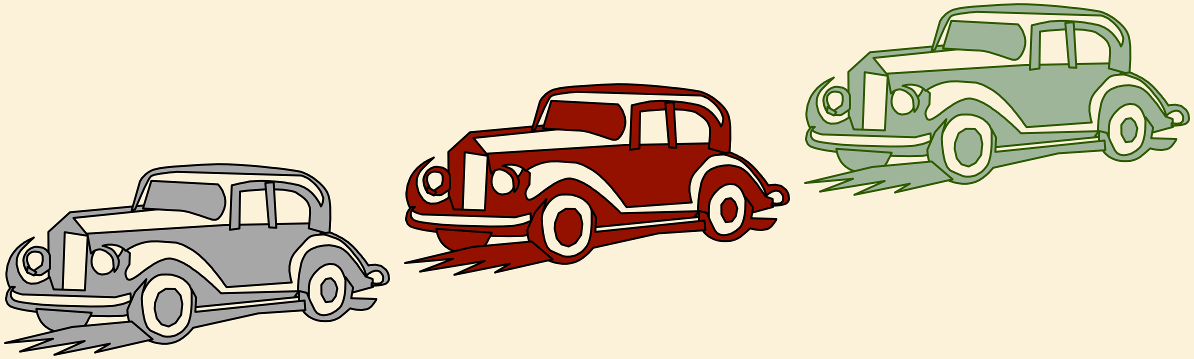
- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element

```
Algorithm size()
   return t + 1

Algorithm pop()
   if isEmpty() then
      throw EmptyStackException
    else
      t ← t - 1
      return S[t + 1]
```



Queues

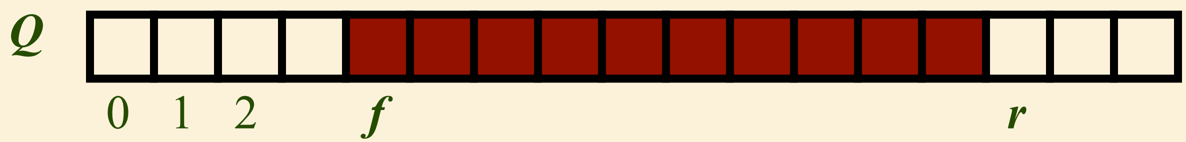
Chapters 5.2-5.3



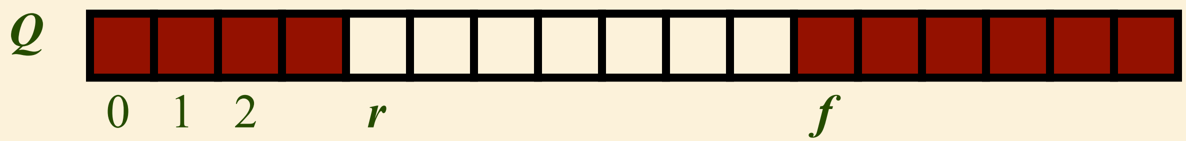
Array-Based Queue

- Use an array of size N in a circular fashion
- Two variables keep track of the front and rear
  - f index of the front element
  - r index immediately past the rear element
- Array location r is kept empty

normal configuration



wrapped-around configuration



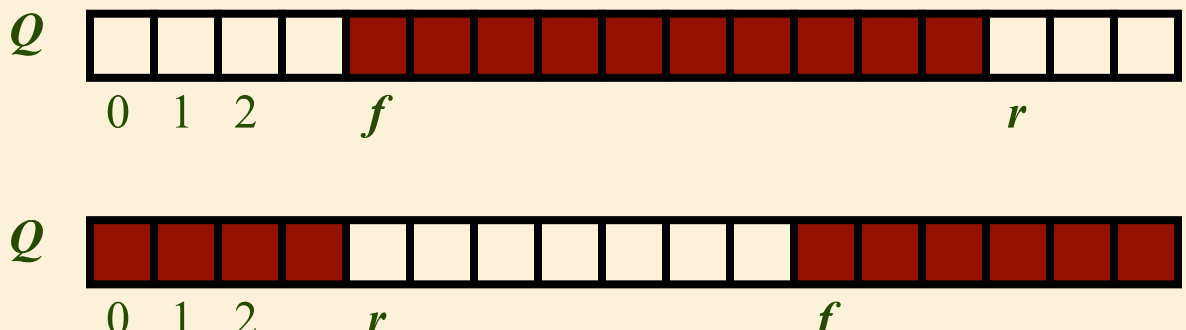
Queue Operations

- We use the modulo operator (remainder of division)

Algorithm *size*()
  return $(N - f + r) \bmod N$

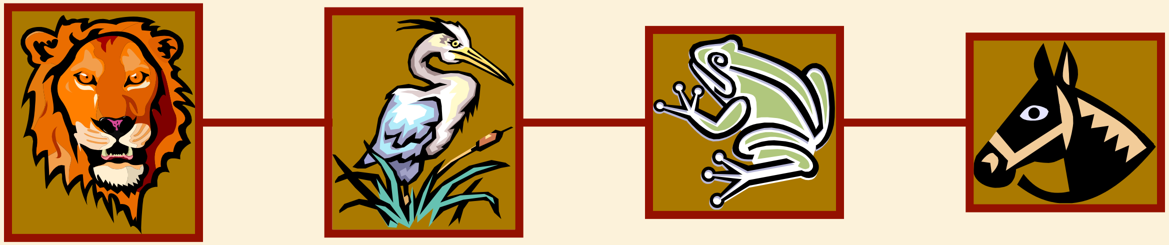
Algorithm *isEmpty*()
  return $(f = r)$

Note: $N - f + r = (r + N) - f$



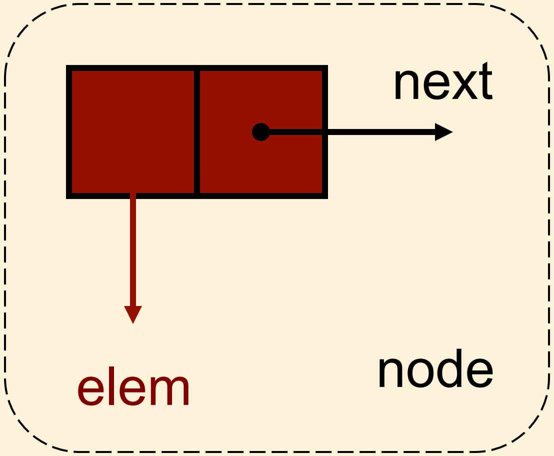
Linked Lists

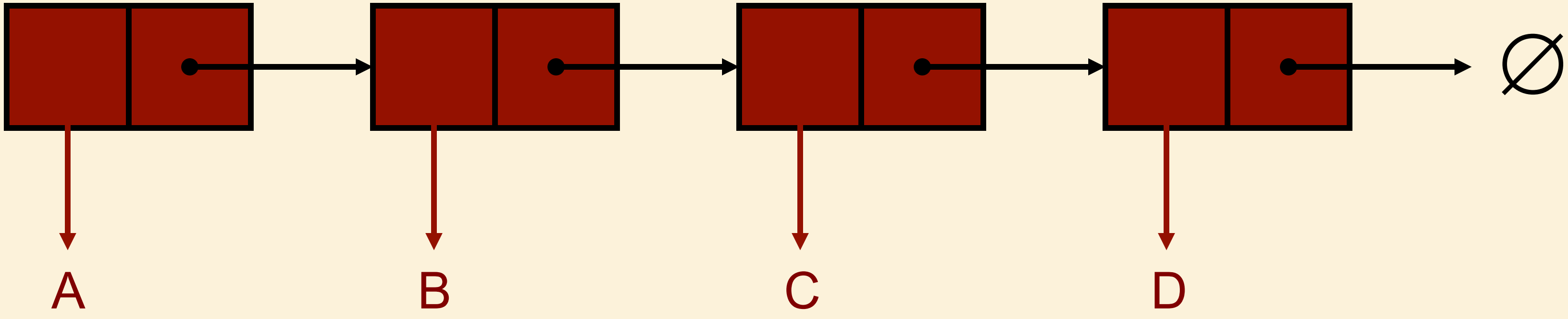
Chapters 3.2 – 3.3



Singly Linked List (§ 3.2)

- A singly linked list is a concrete data structure consisting of a sequence of nodes
- Each node stores
  - element
  - link to the next node



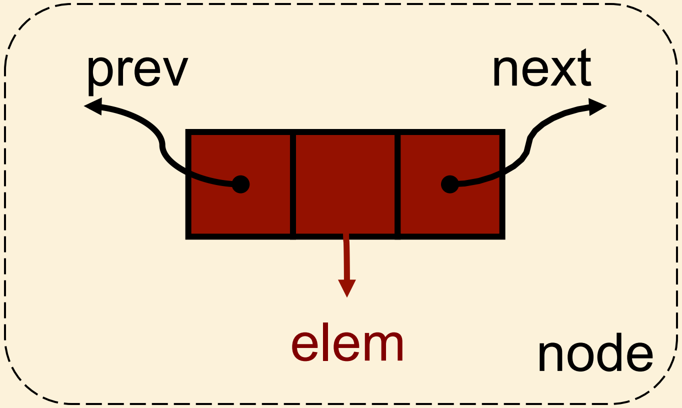


Running Time

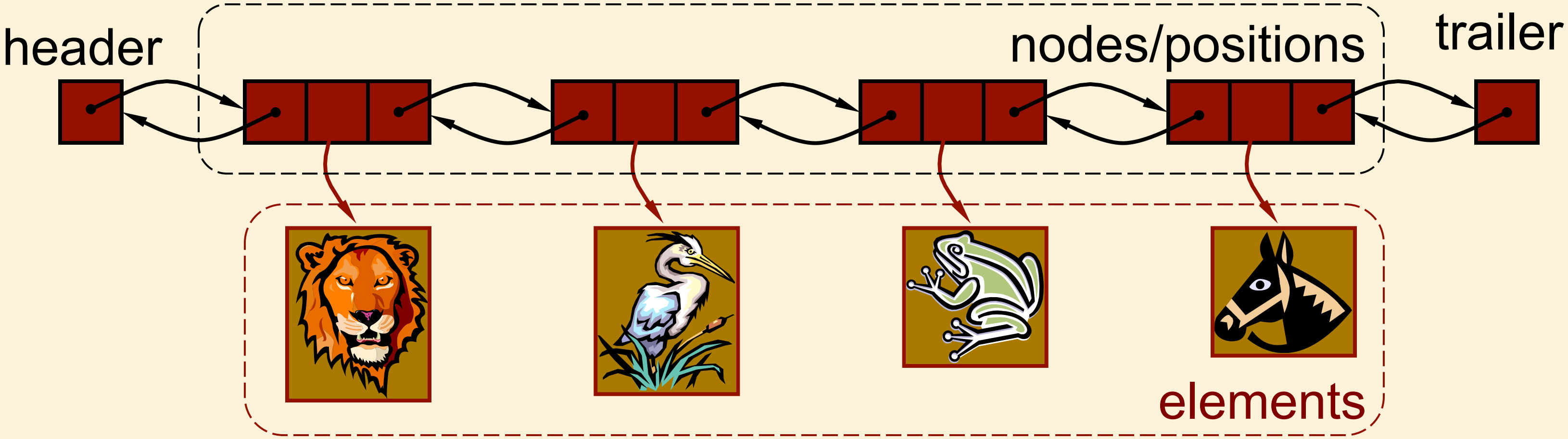
- Adding at the head is O(1)
- Removing at the head is O(1)
- **How about tail operations?**

Doubly Linked List

- Doubly-linked lists allow more flexible list management (constant time operations at both ends).
- Nodes store:
  - element
  - link to the previous node
  - link to the next node



- Special trailer and header (sentinel) nodes



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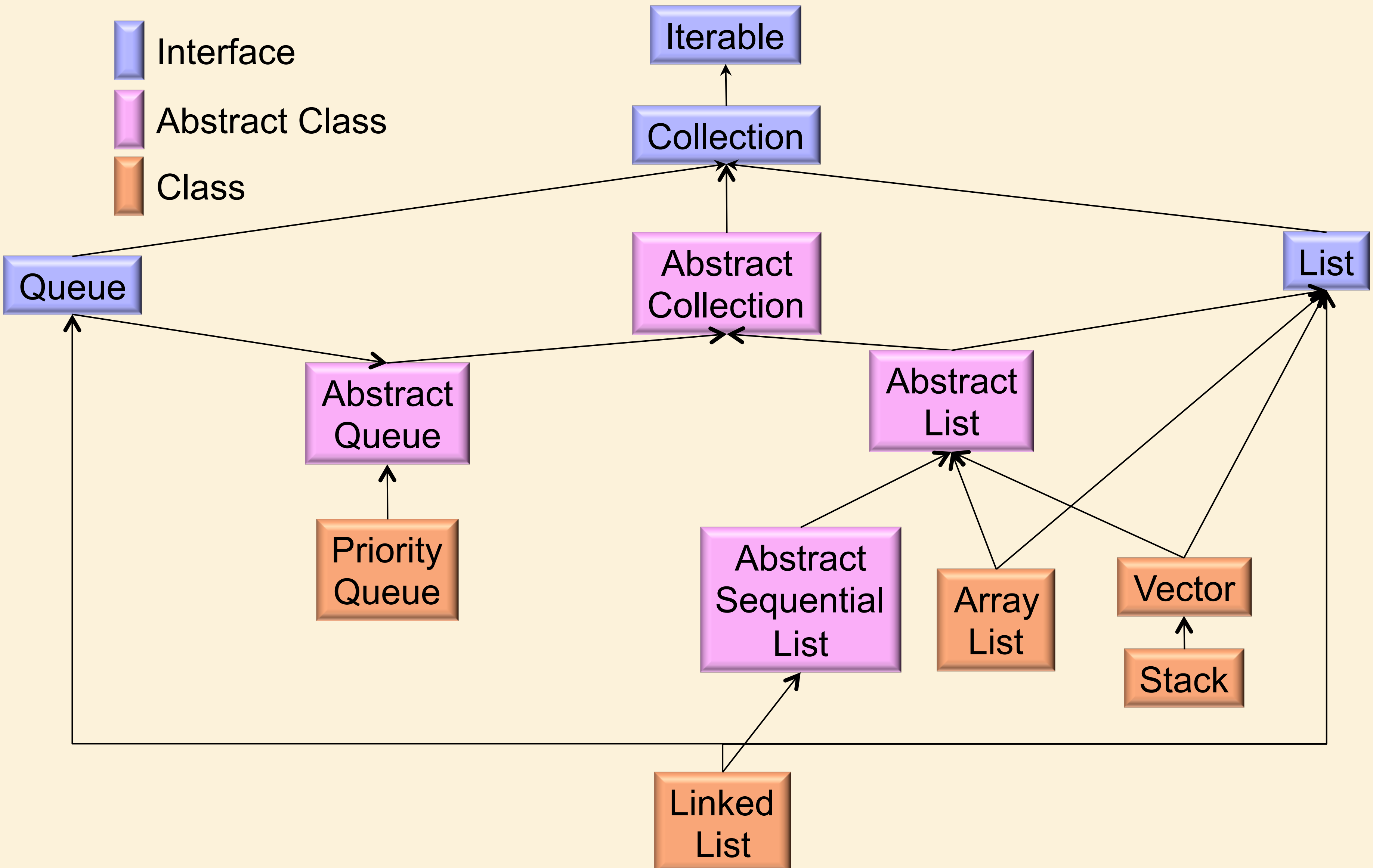
Iterators

- An Iterator is an object that enables you to traverse through a collection and to remove elements from the collection selectively, if desired.
- You get an Iterator for a collection by calling its iterator method.
- Suppose **collection** is an instance of a **Collection**. Then to print out each element on a separate line:

```
Iterator<E> it = collection.iterator();
while (it.hasNext())
    System.out.println(it.next());
```

The Java Collections Framework (Ordered Data Types)



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Linear Recursion Design Pattern

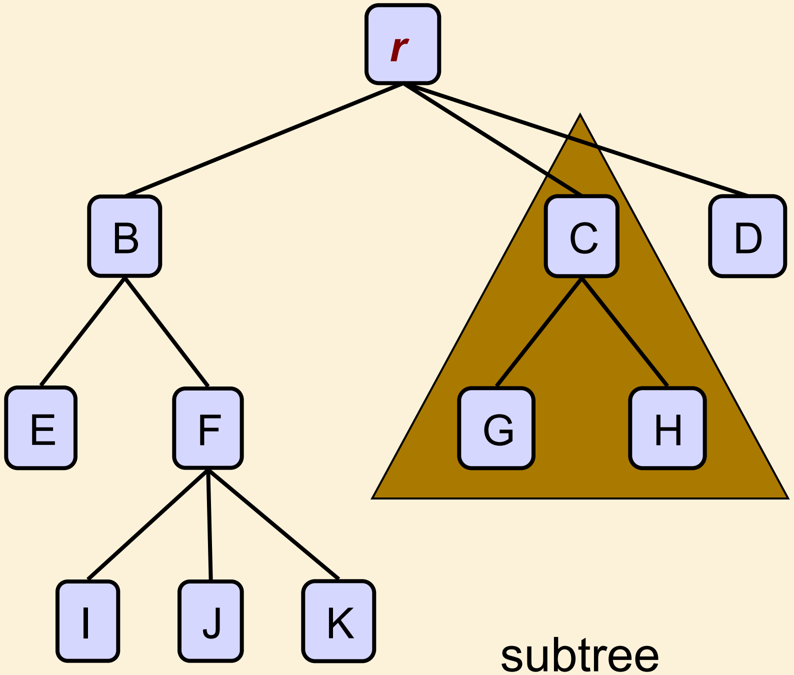
- **Test for base cases**
  - Begin by testing for a set of base cases (there should be at least one).
  - Every possible chain of recursive calls **must** eventually reach a base case, and the handling of each base case should not use recursion.
- ***Recurse once***
  - Perform a single recursive call. (This recursive step may involve a test that decides which of several possible recursive calls to make, but it should ultimately choose to make just one of these calls each time we perform this step.)
  - Define each possible recursive call so that it makes **progress** towards a base case.

Binary Recursion

- Binary recursion occurs whenever there are **two** recursive calls for each non-base case.
- Example 1: **The Fibonacci Sequence**

Formal Definition of Rooted Tree

- A rooted tree may be empty.
- Otherwise, it consists of
  - A root node ***r***
  - A set of **subtrees** whose roots are the children of ***r***



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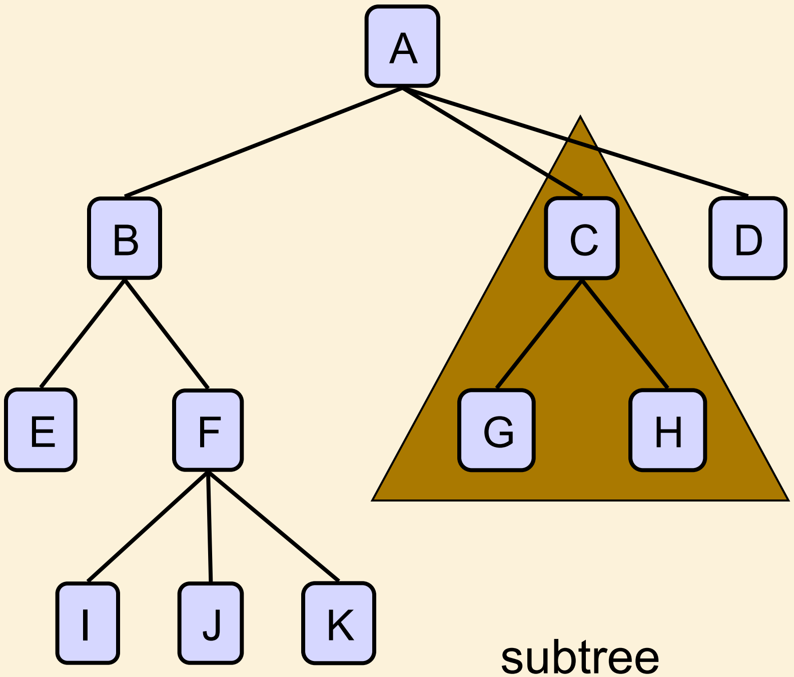
Tree Terminology

- **Root:** node without parent (A)
- **Internal node:** node with at least one child (A, B, C, F)
- **External node (a.k.a. leaf):** node without children (E, I, J, K, G, H, D)
- **Ancestors of a node:** parent, grandparent, grand-grandparent, etc.
- **Descendant of a node:** child, grandchild, grand-grandchild, etc.
- **Siblings**: two nodes having the same parent
- **Depth of a node:** number of ancestors (excluding self)
- **Height of a tree:** maximum depth of any node (3)
- **Subtree:** tree consisting of a node and its descendants



Position ADT

- The Position ADT models the notion of place within a data structure where a single object is stored
- It gives a unified view of diverse ways of storing data, such as
  - a cell of an array
  - a node of a linked list
  - a node of a tree
- Just one method:
  - object element(): returns the element stored at the position

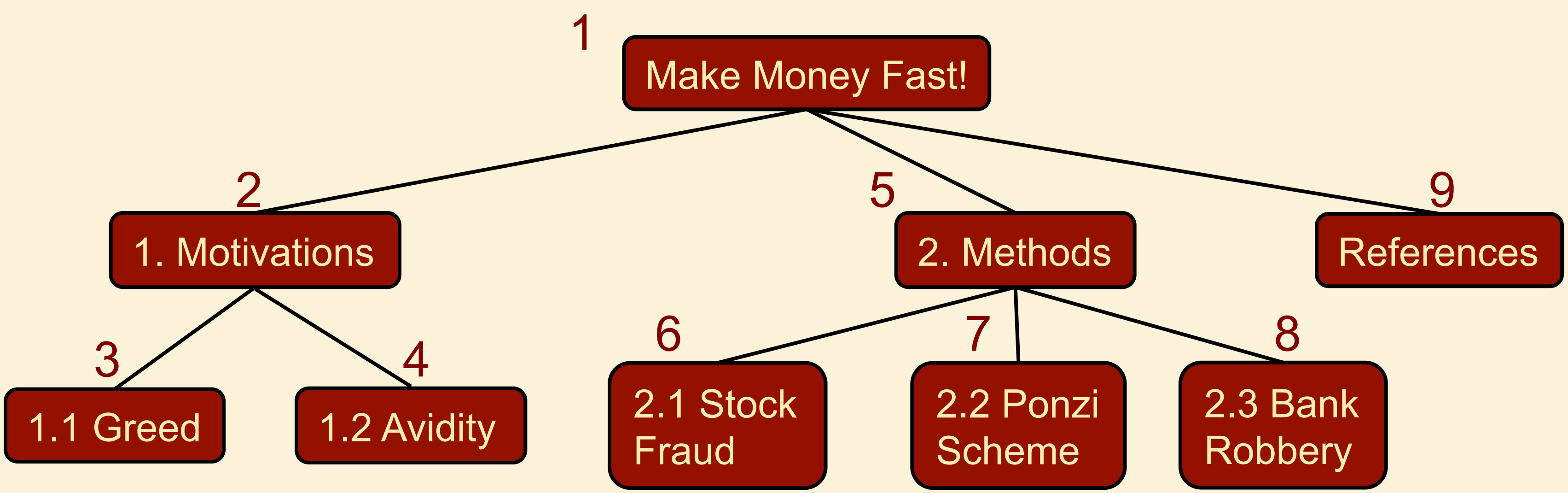
Tree ADT

- We use positions to abstract nodes
- Generic methods:
  - integer size()
  - boolean isEmpty()
  - Iterator iterator()
  - Iterable positions()
- Accessor methods:
  - position root()
  - position parent(p)
  - positionIterator children(p)
- Query methods:
  - boolean isInternal(p)
  - boolean isExternal(p)
  - boolean isRoot(p)
- Update method:
  - object replace(p, o)
  - Additional update methods may be defined by data structures implementing the Tree ADT

Preorder Traversal

- A traversal visits the nodes of a tree in a systematic manner
- In a preorder traversal, a node is visited before its descendants

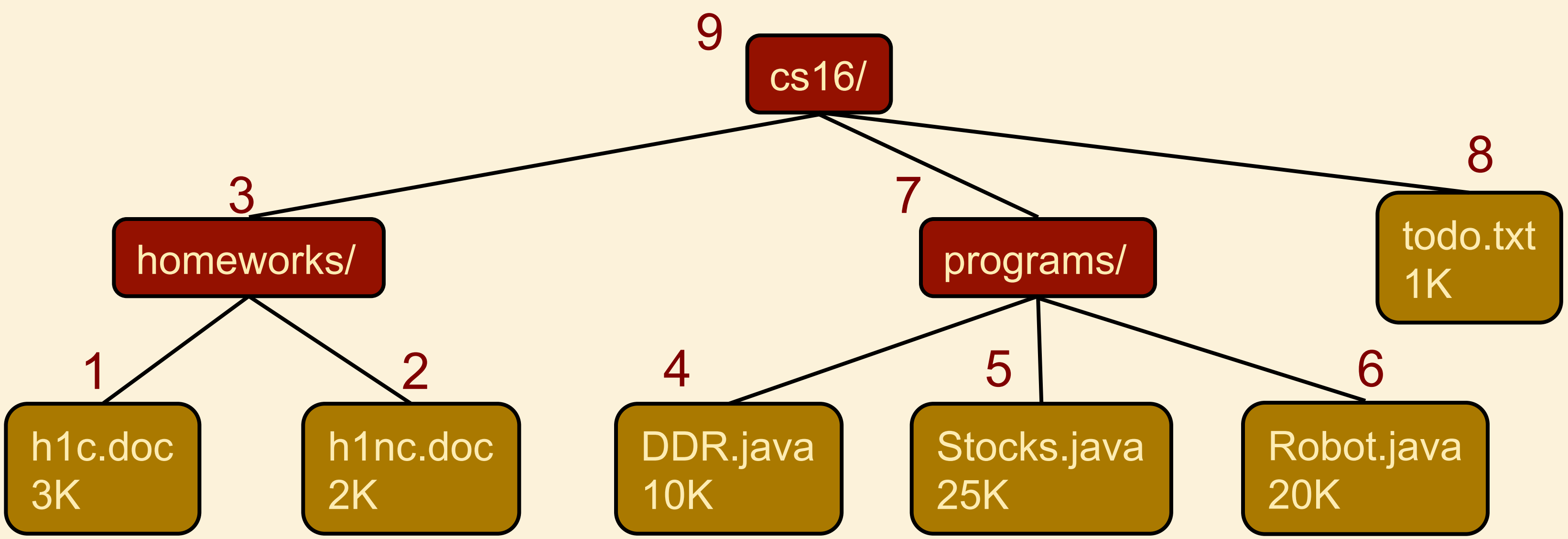
```
Algorithm preOrder(v)
    visit(v)
    for each child w of v
        preOrder (w)
```



Postorder Traversal

- In a postorder traversal, a node is visited after its descendants

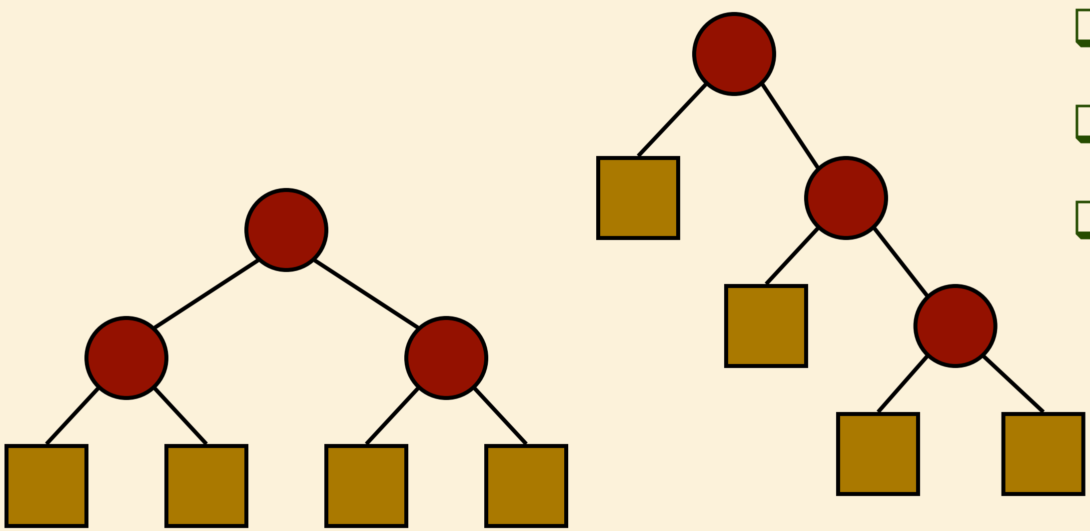
Algorithm ***postOrder***(***v***)
  for each child ***w*** of ***v***
    postOrder (***w***)
  visit(***v***)



Properties of Proper Binary Trees

- Notation
  - ***n*** number of nodes
  - ***e*** number of external nodes
  - ***i*** number of internal nodes
  - ***h*** height

- Properties:
  - ❑ $e = i + 1$
  - ❑ $n = 2e - 1$
  - ❑ $h \leq i$
  - ❑ $h \leq (n - 1)/2$
  - ❑ $e \leq 2^h$
  - ❑ $h \geq \log_2 e$
  - ❑ $h \geq \log_2(n + 1) - 1$



BinaryTree ADT

- The BinaryTree ADT extends the Tree ADT, i.e., it inherits all the methods of the Tree ADT
- Additional methods:
  - position **left**(p)
  - position **right**(p)
  - boolean **hasLeft**(p)
  - boolean **hasRight**(p)
- Update methods may be defined by data structures implementing the BinaryTree ADT

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Priority Queue ADT

- A priority queue stores a collection of **entries**
- Each **entry** is a pair (key, value)
- Main methods of the Priority Queue ADT
  - **insert**(k, x) inserts an entry with key k and value x
  - **removeMin**() removes and returns the entry with smallest key
- Additional methods
  - **min**() returns, but does not remove, an entry with smallest key
  - **size**(), **isEmpty**()
- Applications:
  - Process scheduling
  - Standby flyers

Entry ADT

- An **entry** in a priority queue is simply a key-value pair
- Methods:
  - **key**(): returns the key for this entry
  - **value**(): returns the value for this entry
- As a Java interface:

```
/**
  * Interface for a key-value
  * pair entry
 **/
public interface Entry {
   public Object key();
   public Object value();
}
```

Comparator ADT

- A comparator encapsulates the action of comparing two objects according to a given total order relation
- A generic priority queue uses an auxiliary comparator
- The comparator is external to the keys being compared
- When the priority queue needs to compare two keys, it uses its comparator
- The primary method of the Comparator ADT:
  - **compare**(a, b):
    - Returns an integer *i* such that
      - $i < 0$ if $a < b$
      - $i = 0$ if $a = b$
      - $i > 0$ if $a > b$
      - an error occurs if *a* and *b* cannot be compared.

Sequence-based Priority Queue

- Implementation with an unsorted list

- Performance:
  - **insert** takes $O(1)$ time since we can insert the item at the beginning or end of the sequence
  - **removeMin** and **min** take $O(n)$ time since we have to traverse the entire sequence to find the smallest key

- Implementation with a sorted list

- Performance:
  - **insert** takes $O(n)$ time since we have to find the right place to insert the item
  - **removeMin** and **min** take $O(1)$ time, since the smallest key is at the beginning

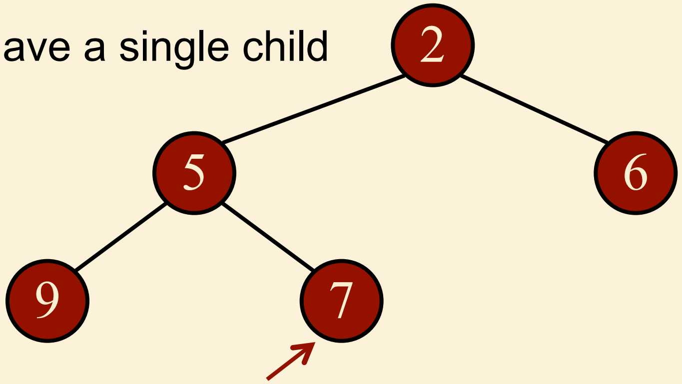
Is this tradeoff inevitable?

Heaps

- Goal:
  - O(log n) insertion
  - O(log n) removal
- Remember that O(log n) is almost as good as O(1)!
  - e.g., n = 1,000,000,000 → log n ≅ 30
- There are min heaps and max heaps. We will assume min heaps.

Min Heaps

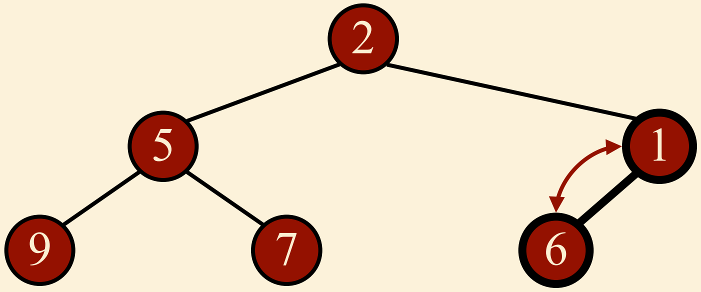
- A min heap is a binary tree storing keys at its nodes and satisfying the following properties:
  - Heap-order: for every internal node v other than the root
    - $key(v) \geq key(parent(v))$
  - (Almost) complete binary tree: let h be the height of the heap
    - for $i = 0, \ldots, h-1$, there are 2^i nodes of depth i
    - at depth $h-1$
      - the internal nodes are to the left of the external nodes
      - Only the rightmost internal node may have a single child

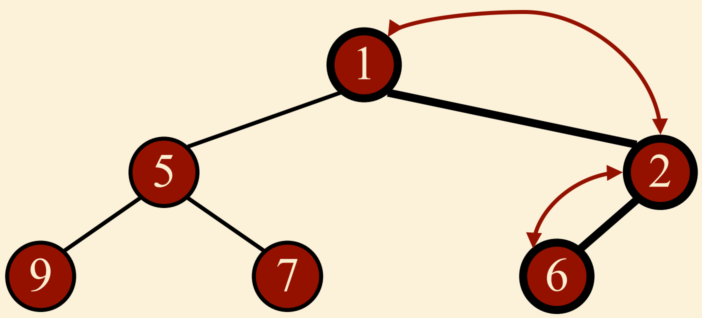


- **The last node of a heap is the rightmost node of depth h**

Upheap

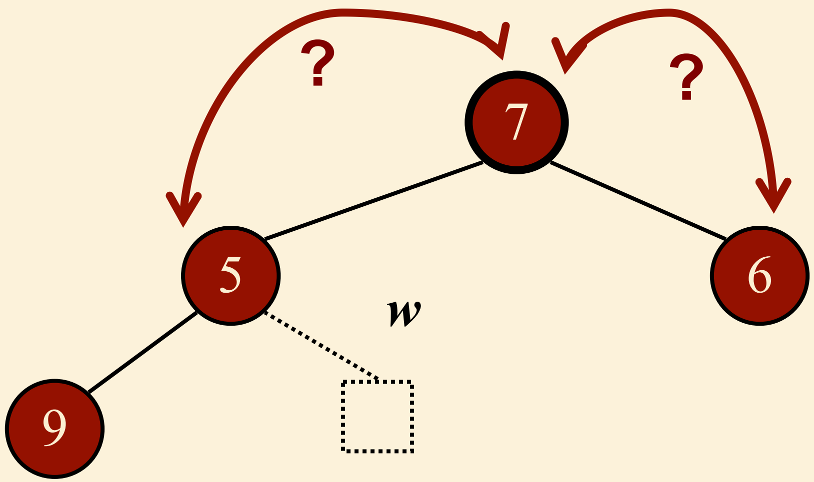
- After the insertion of a new key k, the heap-order property may be violated
- Algorithm **upheap** restores the heap-order property by swapping k along an upward path from the insertion node
- **Upheap** terminates when the key k reaches the root or a node whose parent has a key smaller than or equal to k
- Since a heap has height $O(\log n)$, **upheap** runs in $O(\log n)$ time





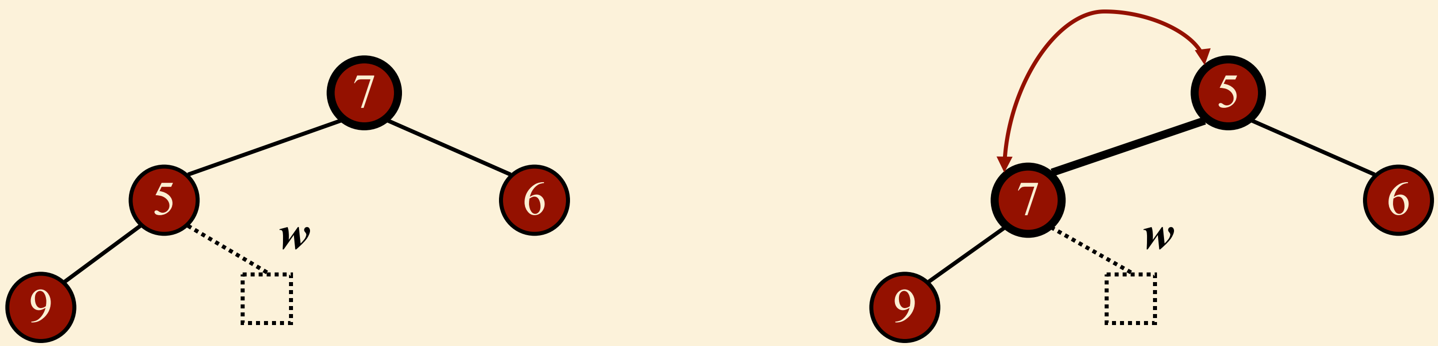
Downheap

- After replacing the root key with the key ***k*** of the last node, the heap-order property may be violated
- Algorithm downheap restores the heap-order property by swapping key ***k*** along a downward path from the root
- Note that there are, in general, many possible downward paths – which one do we choose?



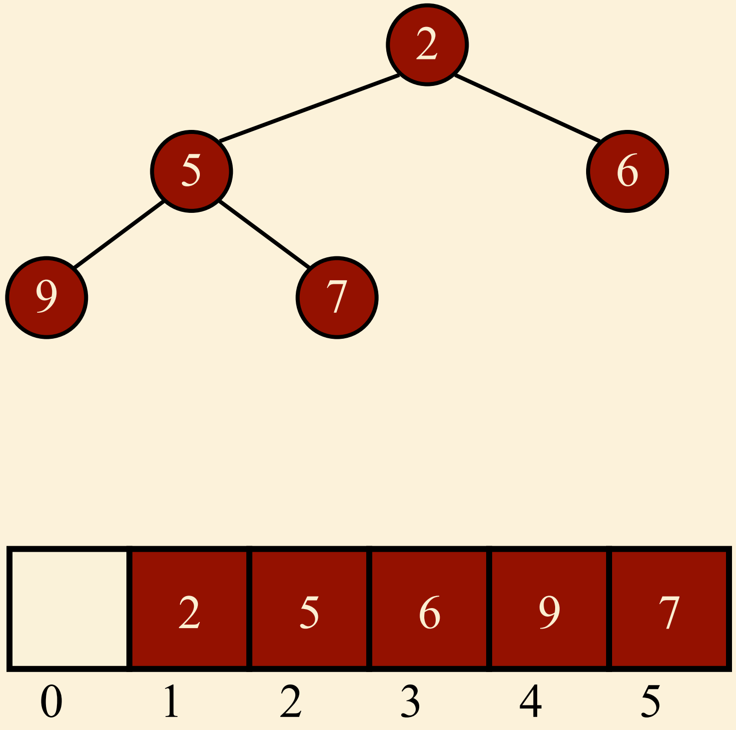
Downheap

- We select the downward path through the **minimum-key** nodes.
- Downheap terminates when key k reaches a leaf or a node whose children have keys greater than or equal to k
- Since a heap has height $O(\log n)$, downheap runs in $O(\log n)$ time



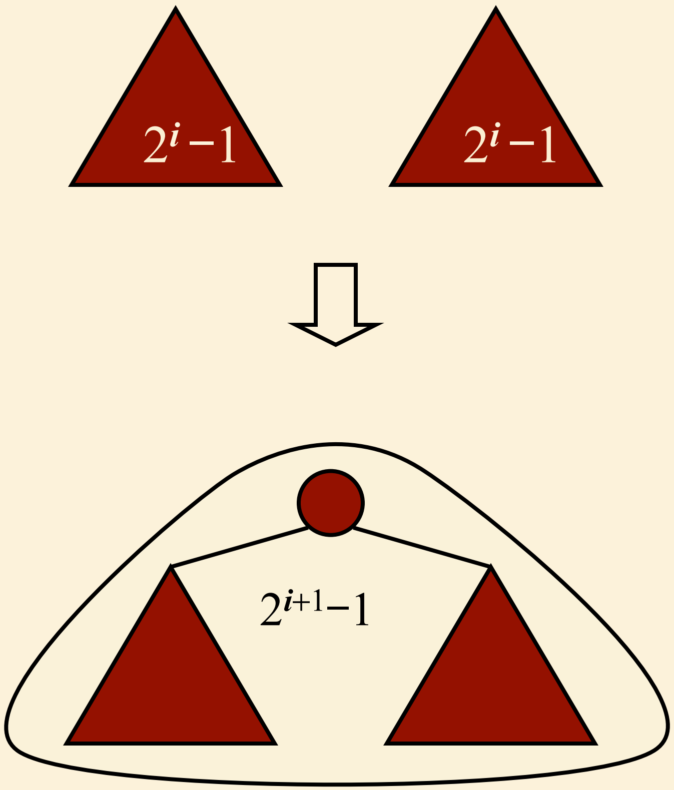
Array-based Heap Implementation

- We can represent a heap with n keys by means of an array of length $n + 1$
- Links between nodes are not explicitly stored
- The cell at rank 0 is not used
- The root is stored at rank 1.
- For the node at rank i
  - the left child is at rank $2i$
  - the right child is at rank $2i + 1$
  - the parent is at rank **floor**(i/2)
  - if 2i + 1 > n, the node has no right child
  - if 2i > n, the node is a leaf

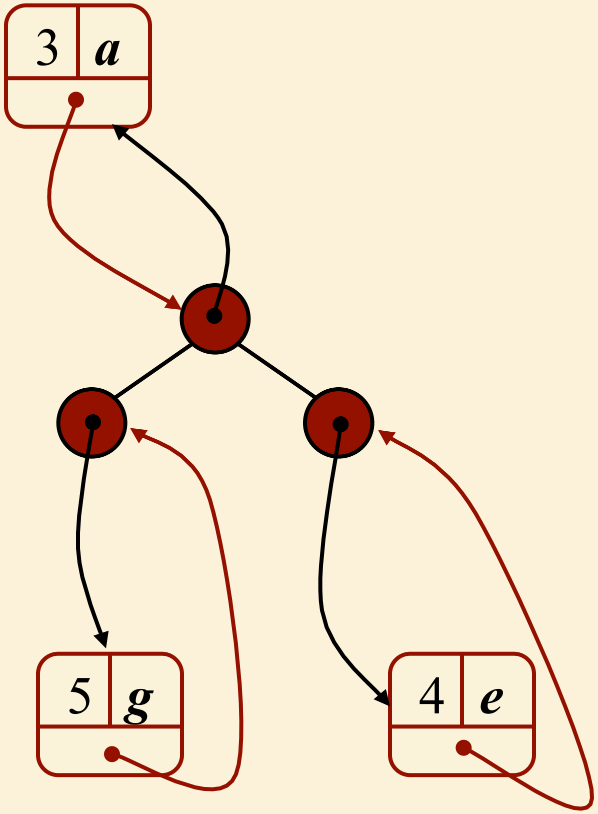


Bottom-up Heap Construction

- We can construct a heap storing n keys using a bottom-up construction with $\log n$ phases
- In phase i, pairs of heaps with $2^i - 1$ keys are merged into heaps with $2^{i+1} - 1$ keys
- Run time for construction is $O(n)$.



Adaptable Priority Queues



Additional Methods of the Adaptable Priority Queue ADT

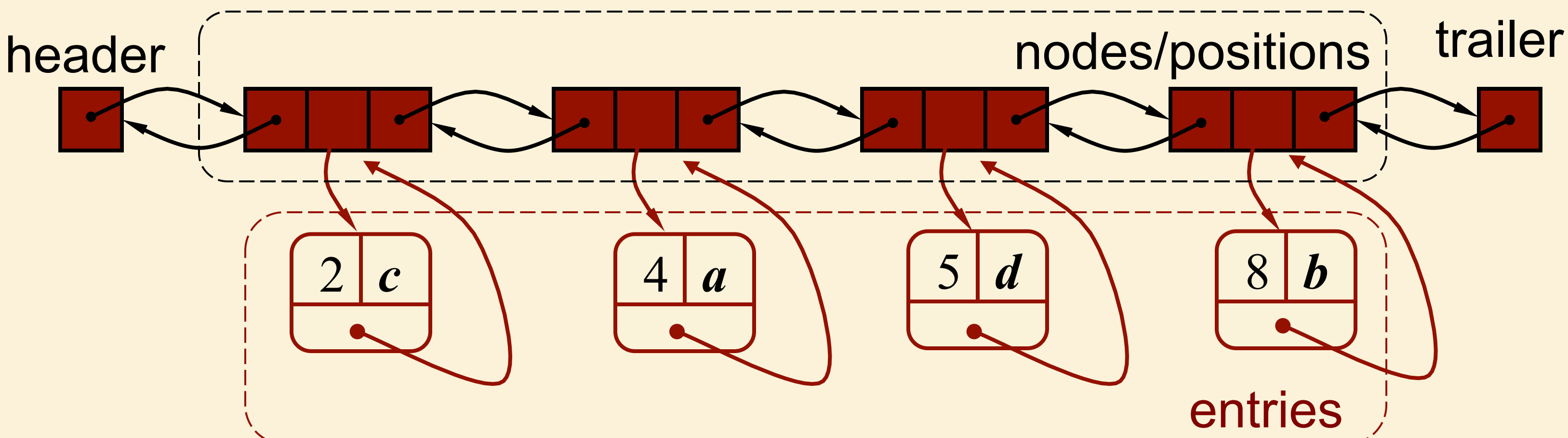
- remove(*e*): Remove from *P* and return entry *e*.
- replaceKey(*e*,*k*): Replace with *k* and return the old key; an error condition occurs if *k* is invalid (that is, *k* cannot be compared with other keys).
- replaceValue(*e*,*x*): Replace with *x* and return the old value.

Location-Aware Entries

- A locator-aware entry identifies and tracks the location of its (key, value) object within a data structure

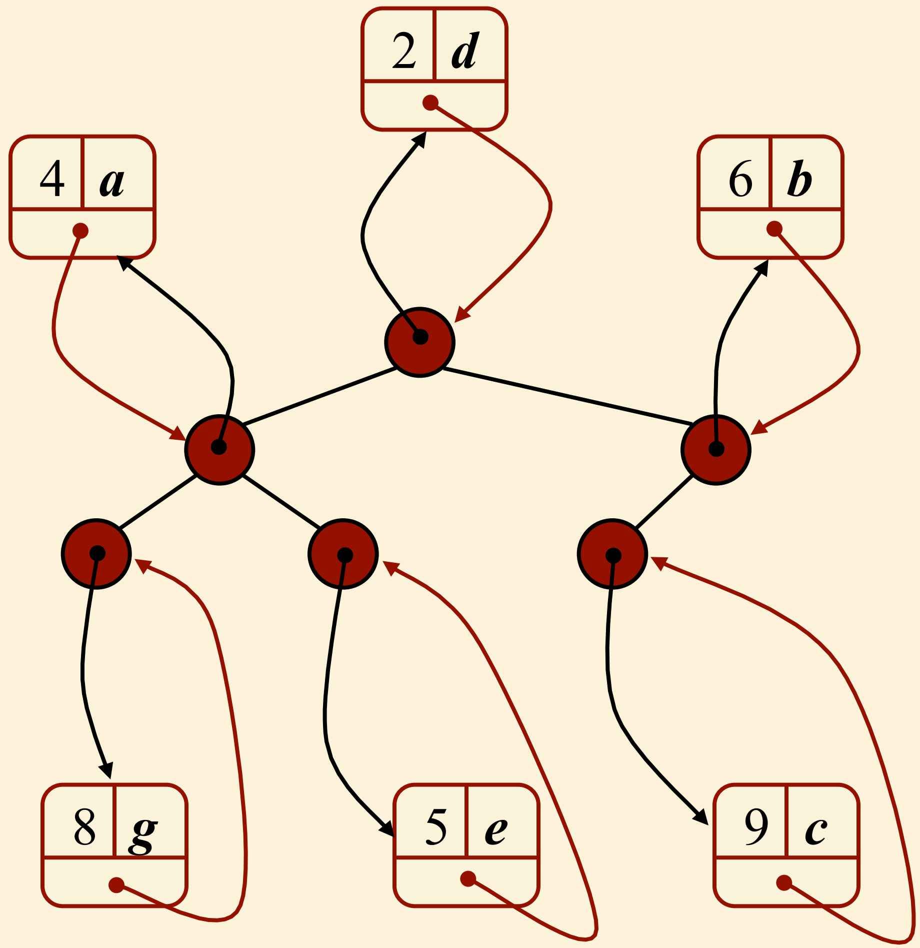
List Implementation

- A location-aware list entry is an object storing
  - key
  - value
  - position (or rank) of the item in the list
- In turn, the position (or array cell) stores the entry
- Back pointers (or ranks) are updated during swaps



Heap Implementation

- A location-aware heap entry is an object storing
  - key
  - value
  - position of the entry in the underlying heap
- In turn, each heap position stores an entry
- Back pointers are updated during entry swaps



Performance

- Times better than those achievable without location-aware entries are highlighted in red:

Method	**Unsorted List**	**Sorted List**	**Heap**
size, isEmpty	$O(1)$	$O(1)$	$O(1)$
insert	$O(1)$	$O(n)$	$O(\log n)$
min	$O(n)$	$O(1)$	$O(1)$
removeMin	$O(n)$	$O(1)$	$O(\log n)$
remove	**$O(1)$**	**$O(1)$**	**$O(\log n)$**
replaceKey	**$O(1)$**	$O(n)$	**$O(\log n)$**
replaceValue	**$O(1)$**	**$O(1)$**	**$O(1)$**

Topics on the Midterm

- Data Structures & Object-Oriented Design
- Run-Time Analysis
- Linear Data Structures
- The Java Collections Framework
- Recursion
- Trees
- Priority Queues & Heaps